
MATHEMATICS FOR A NEW GENERATION OF STUDENTS

Henderson Avenue P.S.

February 03, 2017

Positive Norms to Encourage in Math Class By Jo Boaler

Everyone Can Learn Math to the Highest Levels

- 1. Encourage students to believe in themselves.** There is no such thing as a “math” person. Everyone can reach the highest levels they want to, with hard work.
- 2. Mistakes are Valuable Mistakes grow your brain!** It is good to struggle and make mistakes.
- 3. Questions are Really Important Always ask questions, always answer questions.** Ask yourself: why does that make sense?
- 4. Math is about Creativity and Making Sense** Math is a very creative subject that is, at its core, about visualizing patterns and creating solution paths that others can see, discuss and critique.
- 5. Math is about Connections and Communicating** Math is a connected subject, and a form of communication. Represent math in different forms e.g. words, a picture, a graph, an equation, and link them. Color code!
- 6. Depth is much more important than Speed** Top mathematicians, such as Laurent Schwartz, think slowly and deeply.
- 7. Math Class is about Learning not Performing** Math is a growth subject, it takes time to learn and it is all about effort

Getting Started: Thinking Mathematically

Grades 1 and 2

Today John and Paul added their ages and got 12.

What number will they get in four years?

Grades 3 and 4

A group of 31 students went camping taking nine tents.

Each tent shelters either three or five students.

If all students were covered by a tent, at least how many of the tents housed five students?

Grades 5 and 6

Four points A, B, C, and D are given on a line, in some order, so that $AB=1$, $BC = 2$, $CD = 3$ and $DA = 4$.

Which two points are the farthest from each other?





Canadian Math Kangaroo Contest

Part A: Each correct answer is worth 3 points

1. Michael built a house with matches as in the picture. How many matches did he use?



- (A) 19 (B) 18 (C) 17 (D) 15 (E) 13
2. Which of the following answers gives the largest number less than 15?
 (A) $8+7$ (B) $15-3$ (C) $17-4$ (D) $7+7$ (E) $8+3$

3. How many ropes are there in the picture?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

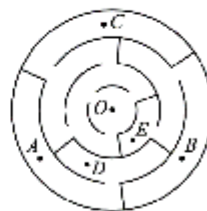


4. Ten friends came to John's birthday party. Six of them were girls. How many boys were there?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

5. Which point of the maze can we reach starting from the point O ?

(A) A (B) B (C) C (D) D (E) E



6. Which year is the next year after the current year, 2016, that is written with the same digits 0, 1, 2 and 6?

(A) 2106 (B) 1206 (C) 2601 (D) 2061 (E) 2160

Problem of the Week [Grades 3 and 4] www.CEMC.UWATERLOO.CA

Sam's Fish Tank

Sam has a large fish tank. He can fit up to 23 fish in his tank.

- ◆ If he groups them in groups of 3, he has 1 fish left.
- ◆ If he groups them in groups of 4, he has 2 fish left.
- ◆ What is the maximum number of fish that Sam could have in his tank?



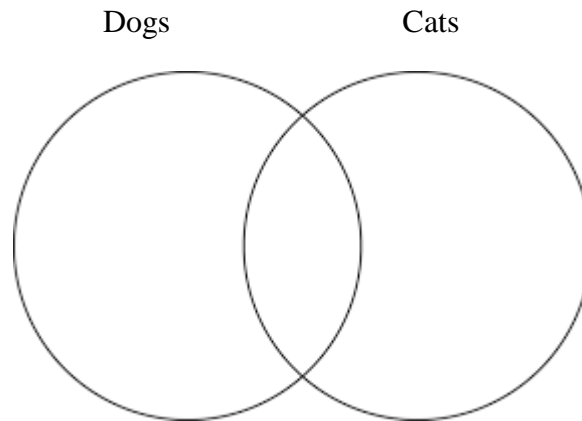
Problem of the Week [Number Sense and Numeration]

There are 26 students in Ms. Scott’s class. They were discussing pets they have at home. When asked, “How many of you have dogs at home?”, 17 students put up their hands.

When asked “How many of you have cats at home?”, 15 students put up their hands.

When asked, “How many of you have both a cat and a dog at home?”, 9 students put up their hands.

How many students have neither cats nor dogs at home? Use a Venn diagram to help you solve this problem.

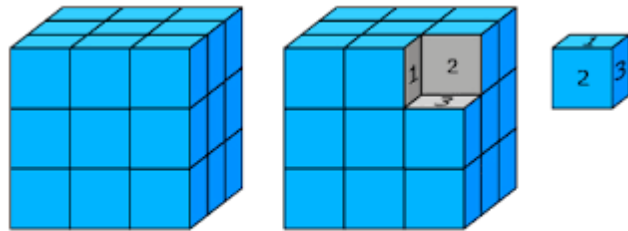


Painted Cube

Imagine a large cube made up from 27 small red cubes.

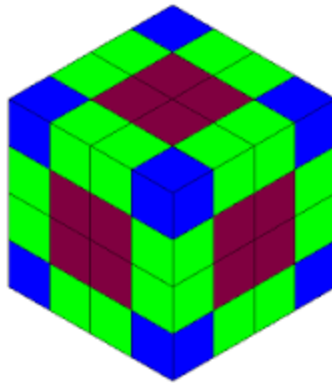
Imagine dipping the large cube into a pot of yellow paint so the whole outer surface is covered, and then breaking the cube up into its small cubes.

How many of the small cubes will have yellow paint on their faces?
Will they all look the same?



When the small cube is separated, it has 3 painted sides. The three sides that were touching the rest of the large cube are not painted.

Now imagine doing the same with other cubes made up from small red cubes. What can you say about the number of small cubes with yellow paint on?



Amy's Dominoes [Number Sense and Numeration]

Amy has a box containing ordinary domino pieces but she does not think it is a complete set.

She has 24 dominoes in her box and there are 125 spots on them altogether.

Which of her domino pieces are missing?



Spatial Sense and Geometry

Need: Interlocking cubes and isometric paper to record answers.

Building Number 1

Clue # 1

There are six cubes in all.

One of the cubes is yellow.

Clue #3

The two red cubes do not touch each other.

Clue #2

The green cube shares one face with each of the other cubes.

Clue #4

The two blue cubes do not touch each other. One of the cubes is yellow.

Building Number 2

Clue #1

There are six cubes in all, in a tower six cubes high.

Clue #3

One of yellows is above the green cube, the other is below it.

Clue #2

The red cube is above the green cube

Clue #4

Each of the blue cubes shares a face with the green cube.

Building Number 3

Clue #1

There is a red cube directly below a yellow cube. There is a green cube on the bottom level.

Clue # 3

There are six cubes in all. An orange cube shares a face with a green cube and two others

Clue #2

There is a red cube on top of a yellow cube. The highest cube is on the third level

Clue #4

A blue cube shares a face with a yellow cube. There is a red cube on the bottom level.

Probability

A magician pulls out pigeons from his hat. Which of the following statements are true, maybe true, or false, if we know that there are 3 gray, 1 white and 2 colorful pigeons in the hat:

- a) if he pull out 4 pigeons, there will be at least one gray among them.
- b) if he pull out 3 pigeons, there will be at least one of each kind of pigeon.
- c) if he pull out 4 pigeons, there will be at least two pigeons of the same color.
- d) if he pull out 6 pigeons, there will be at least two pigeons of each color.
- e) if he pull out 5 pigeons, there will be more colorful than gray.
- f) if he pull out 5 pigeons, there will be more gray than colorful.

Probability

Tony is playing with colored glass balls.

There are 12 red and 8 blue balls in the box.

He takes balls out of the box randomly.

a) How many balls does he have to take out to have two consecutive red balls taken out?

b) How many balls does he have to take out to have two consecutive blue balls taken out?

Mathematics: The Science of Patterns

This is a basic form of the ancient game of Nim.

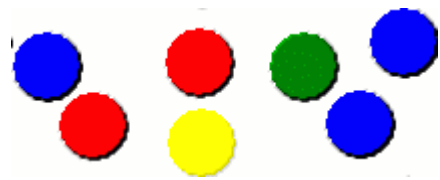
You will need seven objects, such as counters or blocks.

It is a game for two players.

Place the 7 counters in a pile and decide who will go first. (In the next game, the other player will have the first turn.)

Each player takes turns to take away either one or two counters.

The player who takes the last counter wins.




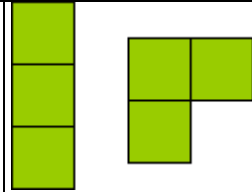
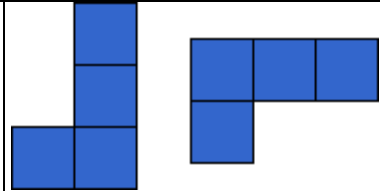
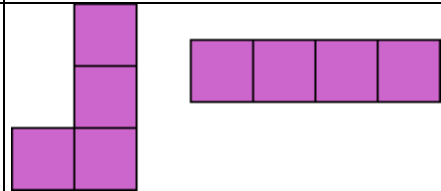
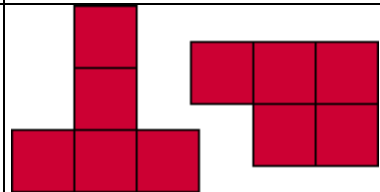
Keep playing until you work out a winning strategy.

Does it matter who has the first turn?

What happens when you start the game with more counters?

Counting Polyominoes

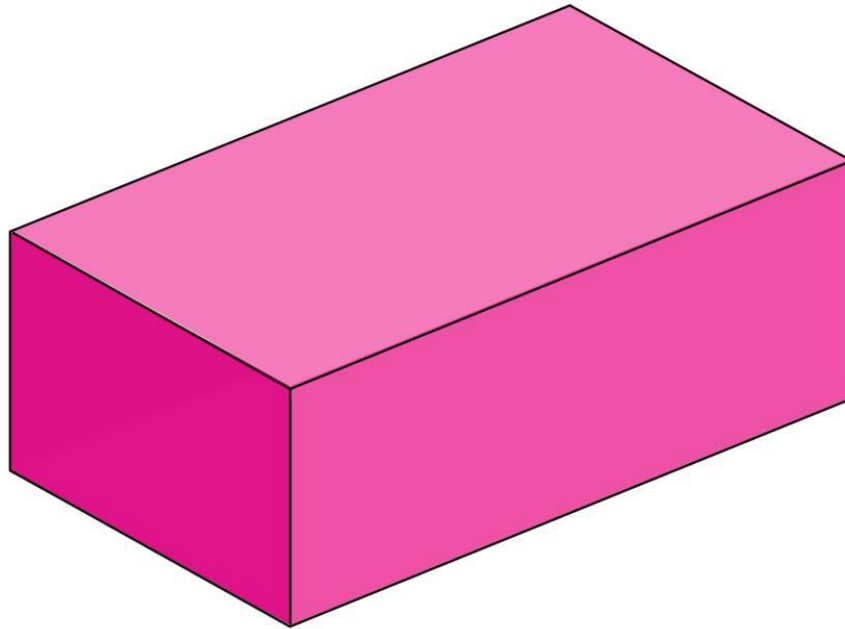
Polyominoes are shapes that are made by gluing identical squares along their edges.

<p>A domino is a shape made from two squares glued along an edge:</p>	
<p>A triomino is a shape made from three squares. There are only two different trominoes:</p>	
<p>A tetromino is a shape made from gluing four squares together. These two blue tetrominoes are actually the same because one can be rotated and flipped on its back to make the other.</p>	
<p>These two tetrominoes are different. How many different tetrominoes can you find?</p>	
<p>A pentomino is a shape made from gluing five squares. The picture on The right shows two different pentominoes. How many different pentominoes are there?</p>	

Problem of the Week {Gauss Math Contest: University of Waterloo}

Measurement Strand

Need: Interlocking Cubes



Tabia has 18 blocks.

Show the number of different solid, rectangular prisms she can build, using all 18 cubes.

Note: A rectangular prism is a 3-D figure with six rectangular faces.

Noah {Number Sense and Numeration}



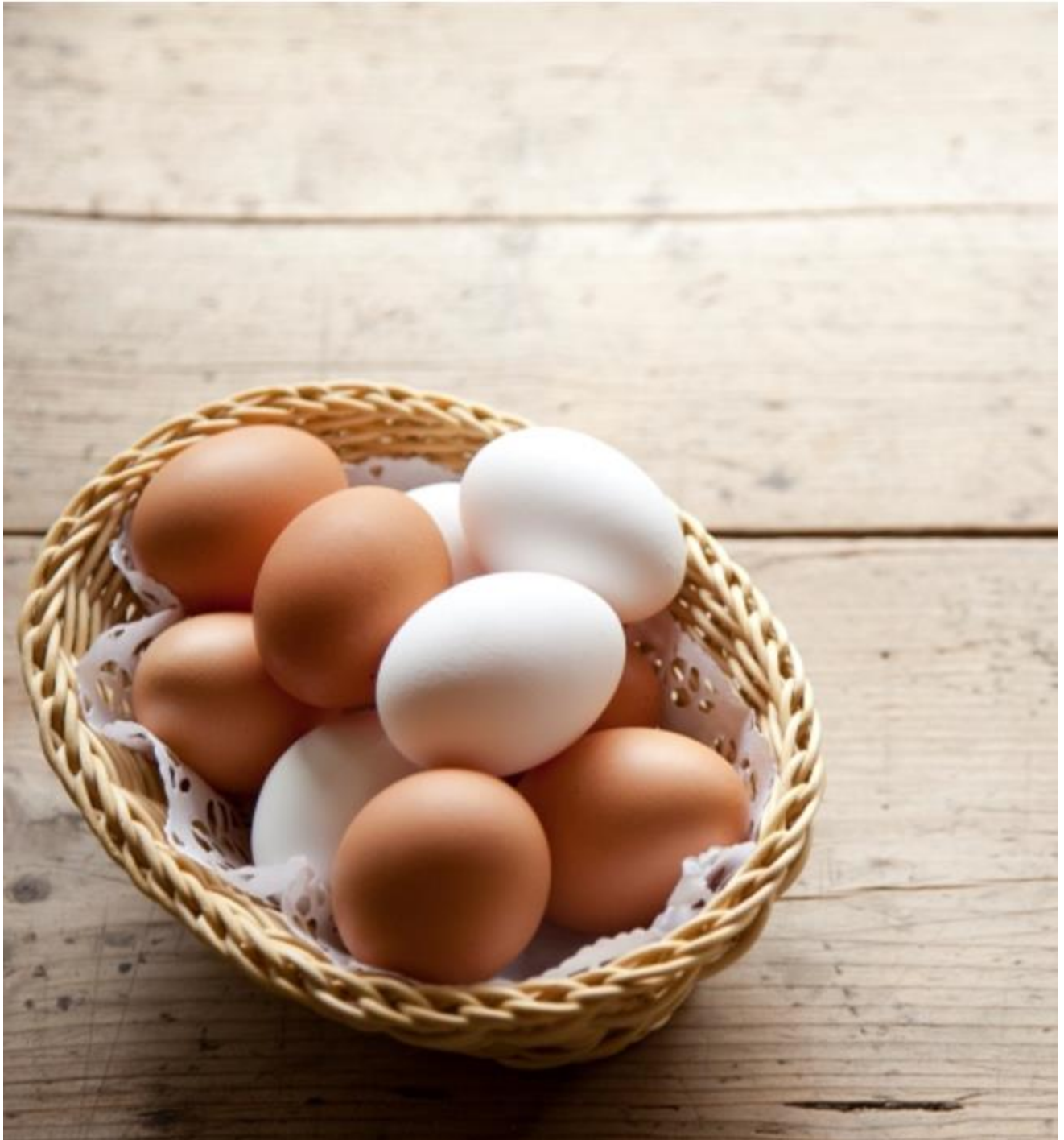
Noah watched the animals going into the ark.

He was counting the legs of the animals and by noon he got to 12.

How many creatures did he see?

See if you can find other answers.

Try to tell someone how you found out these answers.



There are three baskets, a brown one, a red one and a pink one, holding a total of ten eggs. The Brown basket has one more egg in it than the Red basket. The Red basket has three fewer eggs than the Pink basket.

How many eggs are in each basket?

The Prisoners [Data Management and Probability]

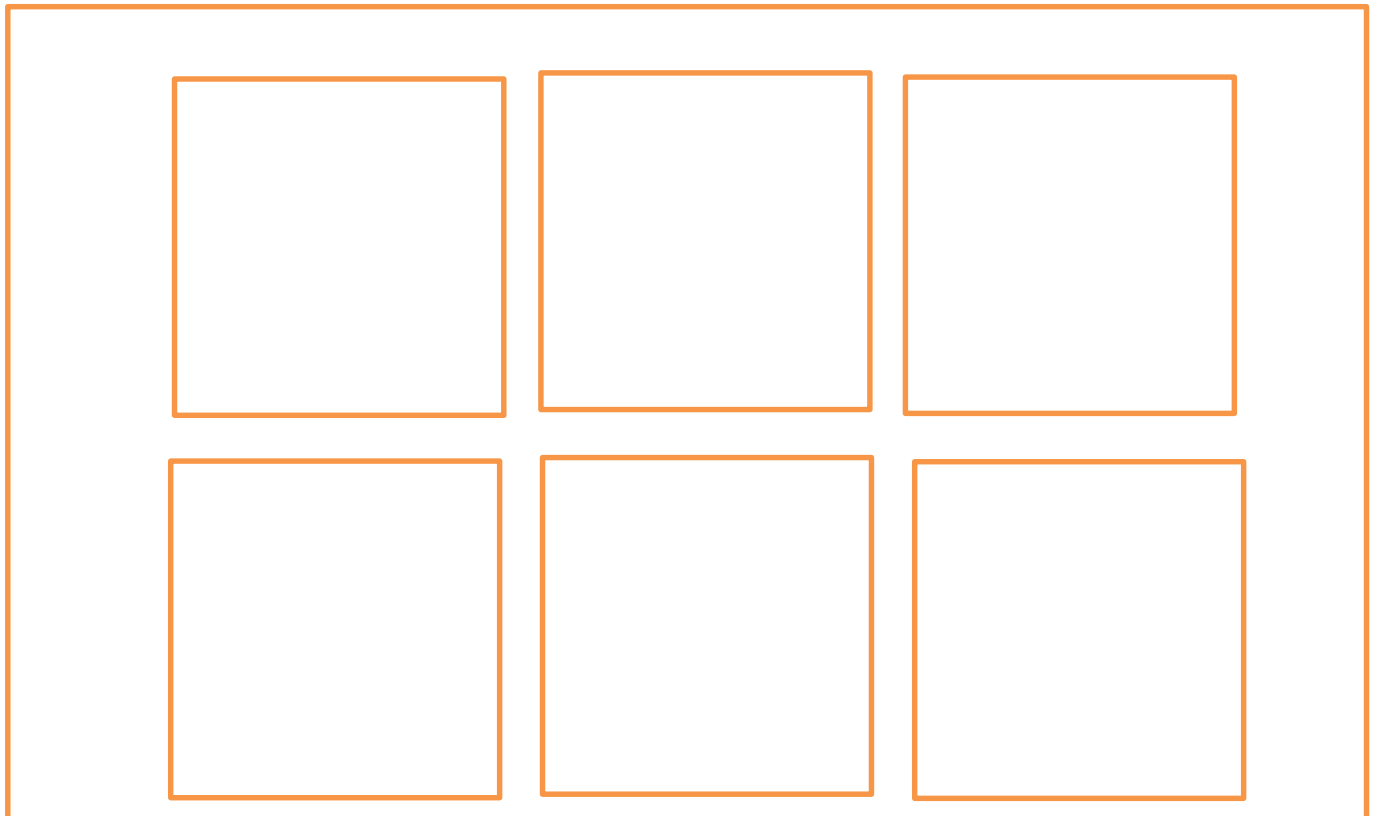
How would you place 6 counters in the cells, so that you can remove them before your partner?

No. of Players: Two

Need: Two Number cubes {Dice}

Rules:

- a) Each player can place his/her counters into any cells on his/her own game board.
- b) You can place one in each cell, or two in some cells and none in others, or all six in one cell.
- c) Take turns to roll the number cubes.
Calculate the difference between the two numbers.
You can remove one counter (only one) from the cell with that number.
- d) The winner is the first to remove all his/her counters from the board.



Tangrams {Spatial Sense and Geometry}

- a) If the area of the small triangle is one square unit, what is the area of the other pieces?
- b) If the value of the square is twenty cents, what is the value of the other pieces?

Making Polygons with Tangram Pieces

	Triangle	Square	Rectangle	Parallelogram	Rhombus	Pentagon
2 pieces						
3 pieces						
4 pieces						
5 pieces						
6 pieces						
7 pieces						
Comments						

Geoboards {Geometry and Spatial Sense}

Need: Geoboards

Part A

Try to make each of the following quadrilaterals {four-sided figures} on your geoboard.

How many of each can you make?

- a) 4 equal sides
- b) 3 equal sides
- c) 2 equal sides
- d) one pair of perpendicular sides
- e) two pairs of perpendicular sides
- f) a right isosceles triangle
- g) a right scalene triangle

Part B

- a) How many different (?) triangles can you make on a geoboard?
- b) How many different {?} quadrilaterals can you make on a geoboard?
- c) How many different sizes of squares can you make on a 5 by 5 geoboard?
- d) How many non-congruent figures can you make with an area of 2 square units?

How Much Can We Spend? {Number Sense and Numeration}

A country has decided to have just two different coins.

It has been suggested that these should be 3z and 5z coins.



The shops think this is a good idea since most totals can be made.



$$2 \times 3z + 1 \times 5z = 11z$$



$$7 \times 3z + 2 \times 5z = 31z$$

Unfortunately some totals can't be made, for example 4z.

Which totals can be made?

Is there a largest total that cannot be made?

How do you know?

They have decided that they will definitely have 3z coins but can't make up their minds about the other coin.

Experiment with other pairings containing $3z$, and explore which totals can be made.

Can you find a relationship between $3z$, the second coin, and the totals that can and can't be made?

In other countries they have also decided to have just two coins, but instead of the $3z$ coins they have chosen a different prime number.

Can you find a relationship between pairs of coin values and the totals that can and can't be made with them?

Medal Muddle {Spatial Sense}

Thirteen nations competed in a sports tournament. Unfortunately, we do not have the final medal table, but we have the following pieces of information:

1. Turkey and Mexico both finished above Italy and New Zealand.
2. Portugal finished above Venezuela, Mexico, Spain and Romania.
3. Romania finished below Algeria, Greece, Spain and Serbia.
4. Serbia finished above Turkey and Portugal, both of whom finished below Algeria and Russia.
5. Russia finished above France and Algeria.
6. Algeria finished below France but above Serbia and Spain.
7. Italy finished below Greece and Venezuela, but above New Zealand.
8. Venezuela finished above New Zealand but below Greece.
9. Greece finished below Turkey, who came below France.
10. Portugal finished below Greece and France.
11. France finished above Serbia, who came above Mexico.
12. Venezuela finished below Mexico, and New Zealand came above Spain.

















Can you recreate the medal table from this information?

Can you describe an efficient strategy for solving problems like this?

What's it worth? {Patterning and Algebra}

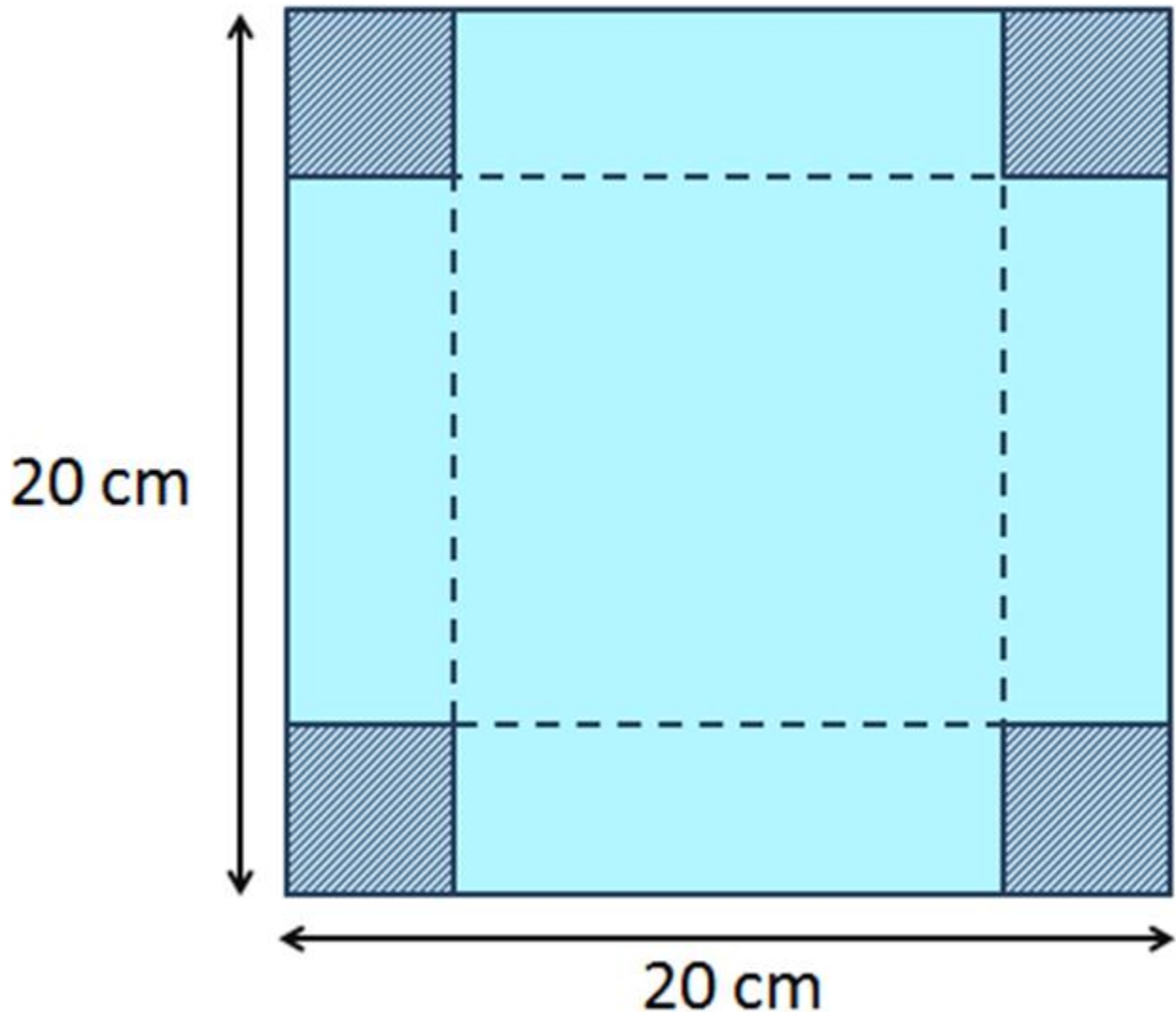
Each symbol has a numerical value. The total for the symbols is written at the end of each row and column.

Can you find the missing total that should go where the question mark has been put?

				28
				30
				18
				20
?	30	23	22	

Cuboid Challenge {Measurement}

From a square sheet of paper 20cm by 20cm, we can make a box without a lid by cutting a square from each corner and folding up the flaps.



Find the different volumes of box it is possible to make by varying the size of the cut-out squares.

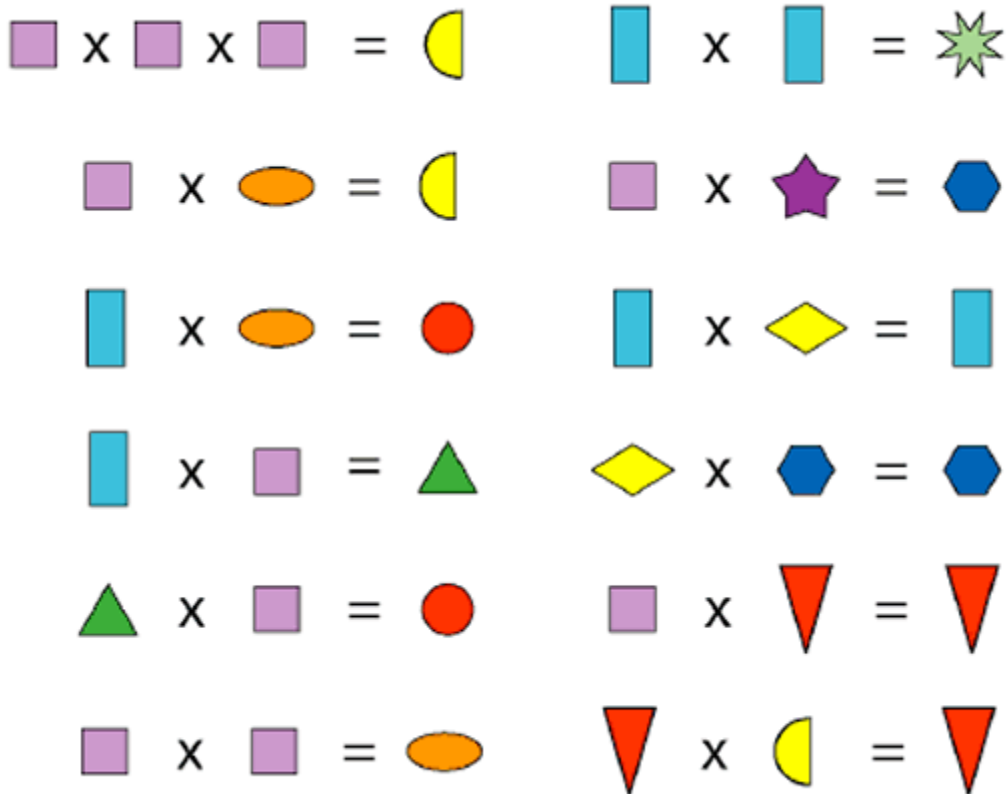
What is the maximum possible volume?

What is the size of the square cut-out that produces it?

Shape Times Shape {Patterning and Algebra}

The shaded shapes stand for eleven of the numbers from 0 to 12. Each shape is a different number.

Can you work out what they are from the multiplications below?



Spatial; Sense Task for Young Mathematicians

Read the clues and put the letters in the grids.

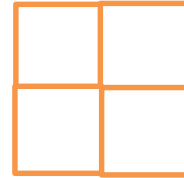
Task 1

A is on the bottom.

B is on the right.

C is under D.

D...you know where it is!



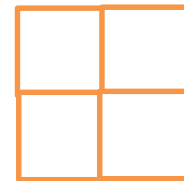
Task 2

A is not on the left.

B is not under C.

C is on the bottom, on the right.

D is on top.



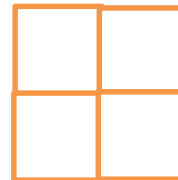
Task 3

A is under C.

B is on the bottom.

C...I forgot...

D is on the left.



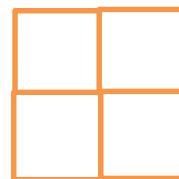
Task 4

A is on the right.

B is on the left.

C is on top.

D is under A.



Task 5

A is in a corner.

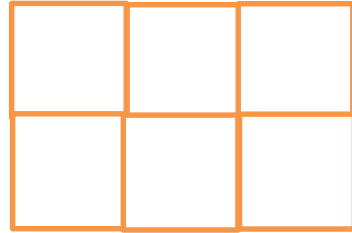
B is between D and F.

C is touching F.

D is touching A.

E is on top, on the left.

F is on the left.



Task 6

A is under E.

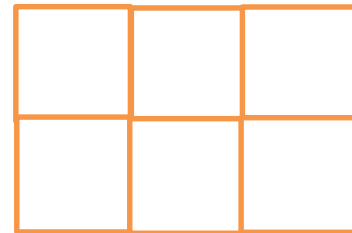
B is in a corner on top.

C is not touching E.

D is on the bottom, but not on the left.

E is on the right.

F is on top.



Task 7

A is touching E.

B is not in the middle.

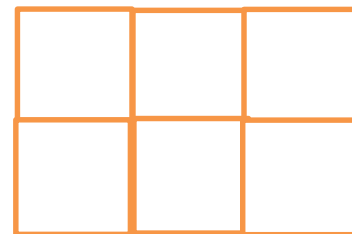
C is between A and F.

D is on the right.

E is next to D.

F is on the bottom.

Can you figure out two different solutions?



Task 8

A is in a corner on the bottom.

B is to the left of G.

C is next to H.

D is in the centre.

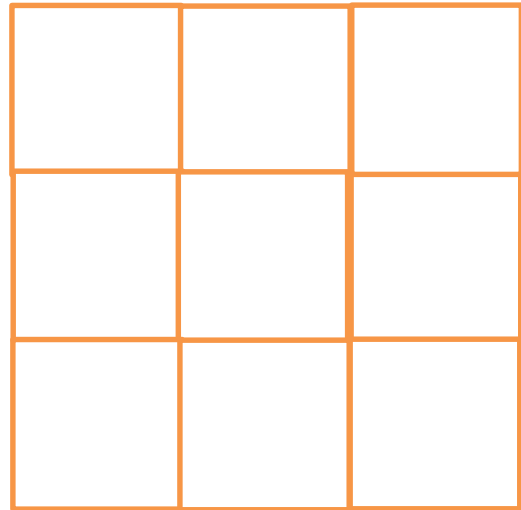
E is touching D.

F is under E.

G is under D.

H is on top.

I is between A and C.



Task 9

A is touching C and D.

B is under H.

C is on the left.

D is on the bottom, on the right.

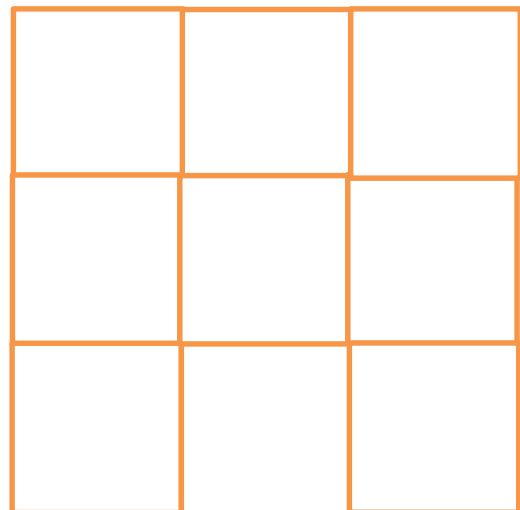
E is next to H.

F is not touching D.

G is in the centre.

H is on the left.

I is touching D.



Biscuit Decorations {Number Sense and Numeration}

Andrew decorated 20 biscuits to take to a party.

He lined them up and put icing on every second biscuit.

Then he put a cherry on every third biscuit.

Then he put a chocolate button on every fourth biscuit.

So there was nothing on the first biscuit.

How many other biscuits had no decoration? Did any biscuits get all three decorations?

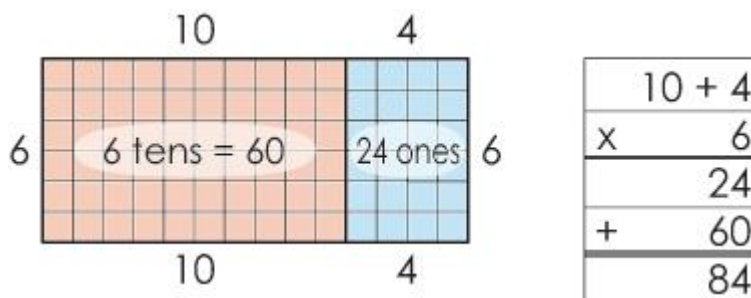


2-digit × 1-digit Multiplication Arrays {Base Ten Materials}

Begin with 2-digit × 1-digit problems. Gauge your students' understanding in this activity to determine the pacing for the 2-digit × 2-digit activity.

Work through the example problem 14×6 to introduce the solution method. Before you begin showing students arrays, have them estimate the answer. This helps develop mental math skills and gives a benchmark to judge accuracy when they complete the problem. Since 14 is closer to 10 than 20, approximate the answer to be about $10 \times 6 = 60$. Since we rounded down, the actual answer will be a bit bigger than 60.

Model the partial product arrays for students using the 14×6 example. Model the following steps, using the illustration as a guide:



- Decompose the first factor into tens and ones to give $(10 + 4) \times 6$. The problem can now be solved using the distributive property.
- Draw an array that is 14×6 .
- Break the 14-unit horizontal line into a tens piece and ones piece. In this case, the pieces will be 10 and 4. Draw a vertical line through the rectangle at this point, creating two rectangles. Label the length and width of both rectangles. Label the 10×6 rectangle as 60, and the 4×6 rectangle as 24, as shown.
- The rectangles represent 2 partial products. Ask the students what to do with the 60 and 24. It is important that students know that *partial* means each array represents just part of the problem. They should also understand that to get the total product you need to add. The solution is $60 + 24 = 84$.

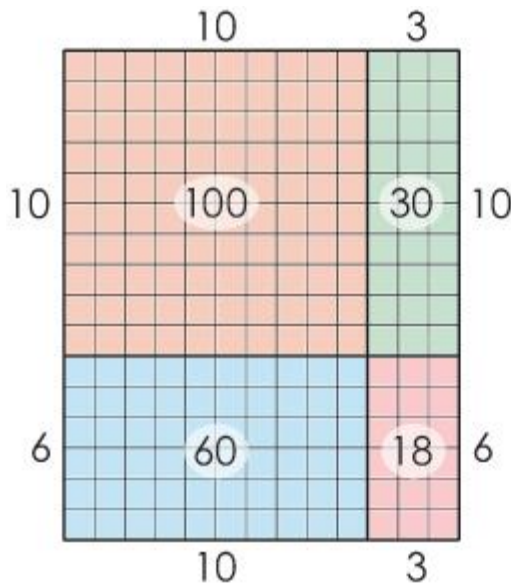
As you model, have students participate by coming up to help build the model. Next, demonstrate solving another problem while students create their own base 10 block or paper grid models.

Have less capable students work with a partner. To transition to more individualized work, have individuals and pairs compare their work with that of others and describe their partial arrays aloud. Do as many problems as needed to support your variety of learners. Keep in mind that large 2-digit numbers get unwieldy to work with; numbers between 10 and 30 work the best for modeling.

2-digit × 2-digit Multiplication Arrays

Model the problem 13×16 . Once students understand the process, they can work problems at your direction for continual practice. Important points for 2-digit by 2-digit multiplication include:

- The problem must be decomposed. It becomes $(10 + 3) \times (10 + 6)$.
- Draw an array that is 13 by 16.
- Decompose the factors by breaking the array into smaller pieces. Break the 13-unit horizontal line into 10- and 3-unit long pieces. Then, break the 16-unit vertical line into 10- and 6-unit long pieces.
- Draw lines to separate the tens from the ones (10 from 3 and 10 from 6). This creates 4 rectangles.
- Label the outside dimensions.
- Students should be able to see the 4 partial products of this problem: (10×10) , (3×10) , (6×10) , and (3×6) .
- Label the inside areas of the 4 partial products.
- Finally, add the partial products together: $100 + 30 + 60 + 18 = 208$.



	10 + 3
x	10 + 6
	18
	60
	30
+	100
	208

Low Stress Long Division {Very Easy!}

Context:

How many packages of 8 cookies if there are 348 cookies to package?

Using just the 10 x, 5 x, 2 x and 1 x tables

$$\begin{array}{r} 8 \overline{) 348} \\ \underline{- 80} \quad 10 \quad \{8 \times 10\} \\ 268 \\ \underline{- 160} \quad 20 \quad \{8 \times 20\} \\ 108 \\ \underline{- 80} \quad 10 \quad \{8 \times 10\} \\ 28 \\ \underline{- 16} \quad 2 \quad \{2 \times 8\} \\ 12 \\ \underline{- 8} \quad 1 \quad \{1 \times 8\} \\ 4 \end{array}$$

43 r 4 {Answer}

Ten Frames

Arranging Colour Tiles

Task #1

The yellow tile is not at the bottom.

The blue tile is not between the green tile and the red tile.

The green tile is between the yellow tile and the blue tile.

The red tile is above the yellow tile.

Task #2

The red tile is somewhere below the yellow tile.

The blue tile is somewhere above the yellow tile.

The top tile is not blue.

The tile below the green tile is red.

Task #3

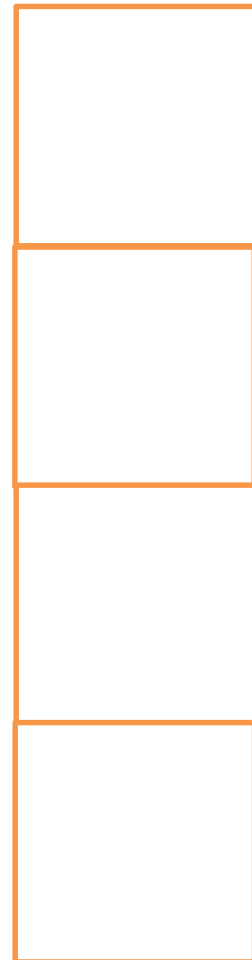
No tiles of the same colour are next to each other.

The yellow tile is not at the bottom.

The blue tile is not between the green tile and the red tile.

The green tile is between the yellow tile and the blue tile.

The red tile is above the yellow tile.



Always, Sometimes or Never? {Assessment Tasks}

Are the following statements always true, sometimes true or never true?

A hexagon has six equal length sides	Triangles have a line of symmetry
Squares have two diagonals that meet at right angles	Cutting a corner off a square makes a pentagon
The base of a pyramid is a square	A cuboid has two square faces

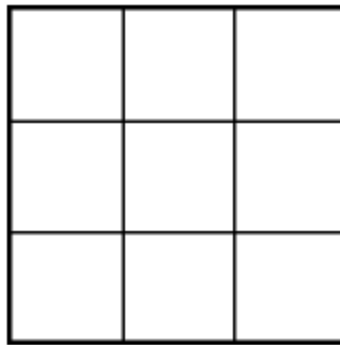
When you cut off a piece from a 2D shape you reduce the area and perimeter	Triangles tessellate
The number of lines of symmetry in a regular shape is equal to the number of sides	Quadrilaterals can be cut into two equal triangles

A Brief History of Magic Squares

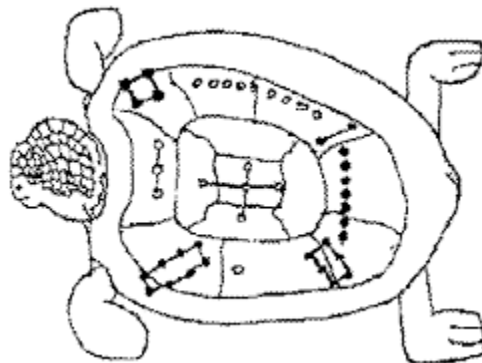
Magic squares have a rich history dating to around 2200 B.C. A Chinese myth claimed that while the Chinese Emperor Yu was walking along the Yellow River, he noticed a tortoise with a unique diagram on its shell (see the picture to the right). The Emperor decided to call the unusual numerical pattern *lo shu*. The earliest magic square on record, however, appeared in the first-century book *Da-Dai Liji*.

Use the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 to form a Magic Square.

The horizontal, vertical, and diagonal sums are the same.



The original Magic Square



A Dickey Situation [Data Management and Probability]

Jamie made up a new dice game.

Two players each roll an ordinary six-sided die.

Of the two numbers showing, the smaller is subtracted from the larger.



Scoring:

- If the difference is 0, 1, or 2, player A gets 1 point.
- If the difference is 3, 4, or 5, player B gets 1 point.

The game ends after 12 rounds.

The player with the most points wins the game.

If you were given the choice of being Player A or Player B, which would you pick, assuming you want to win?

Remember to explain all the steps you use in making your decision.

Extra: Describe another way of scoring that is fair for this game of differences. Explain how you know it is fair.

[A mathematician considers a game to be fair if all players have an equal chance of winning.]